Causal Version of Newtonian Theory by Time–Retardation of the Gravitational Field Explains the Flyby Anomalies

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Classical Newtonian gravitational theory does not satisfy the causality principle because it is based on instantaneous action-at-a-distance. A causal version of Newtonian theory for a large rotating sphere is derived herein by time-retarding the distance between interior circulating point-mass sources and an exterior field-point. The resulting causal theory explains exactly the six flyby anomalies reported by Anderson et al. in 2008. It also explains exactly an anomalous decrease in the Moon's orbital speed. No other known theory has been shown to explain both the flyby anomalies and the lunar orbit anomaly.

1 Introduction

In 2008 Anderson *et al.* reported that anomalous orbital-energy changes have been observed during six spacecraft flybys of the Earth [1]. The reported speed-changes range from a maximum of +13.28 mm/s for the NEAR flyby to a minimum of −4.6 mm/s for the Galileo-II flyby. Anderson *et al.* also found an empirical prediction formula that gives calculated speed-changes that are close to the observed speed-changes. If the speed-change for the empirical prediction formula is designated by δv_{emp}, it can be expressed as follows

$$\delta v_{\text{emp}} = \frac{2\,v_{\text{eq}}}{c}\, v_{\text{in}}\,(\cos\lambda_{\text{in}} - \cos\lambda_{\text{out}}) = = -\frac{2\,v_{\text{eq}}}{c}\, v_{\text{in}} \int_{t_{\text{in}}}^{t_{\text{out}}} \sin(\lambda(t))\,\frac{d\lambda}{dt}\,dt\,, \qquad (1)$$

where v_{eq} is the Earth's equatorial rotational surface speed, c is the vacuum speed of light, v_{in} is the initial asymptotic inbound speed, λ_{in} is the asymptotic inbound geocentric latitude, and λ_{out} is the asymptotic outbound geocentric latitude. If t is the observed coordinate time for the spacecraft in its trajectory, then $\lambda_{\text{in}} = \lambda(t_{\text{in}})$ and $\lambda_{\text{out}} = \lambda(t_{\text{out}})$. If $d\lambda/dt = 0$, then $\delta v_{\text{emp}} = 0$. An order of magnitude estimate for the maximum possible value for δv_{emp} is $2(5\times10^2/3\times10^8)\,v_{\text{in}} \sim 30$ mm/s.

The following is a direct quote from the conclusions of an article published in 2009 by M. M. Nieto and J. D. Anderson [2]:

> "Several physicists have proposed explanations of the Earth flyby anomalies. The least revolutionary invokes dark matter bound to Earth. Others include modifications of special relativity, of general relativity, or of the notion of inertia; a light speed anomaly; or anisotropy in the gravitational field —- all of those, of course, deny concepts that have been well tested. And none of them have made comprehensive, precise predictions of Earth flyby effects. For now the anomalous energy changes observed in Earth flybys remain a puzzle. Are they the result of imperfect understandings of conventional physics and experimental systems, or are they the harbingers of exciting new physics?"

It appears that a new and possibly unconventional theory is needed.

The empirical prediction formula found by Anderson *et al.* is not based on any mainstream theory (it was simply "picked out of the air"), but it is remarkably simple and does produce calculated speed-changes that are surprisingly close to the observed speed-changes. The formula for δv_{emp} (1) gives three clues for properties that need to be satisfied by any theory that is developed to explain the flyby anomaly: 1) the theory must produce a speed-change that is proportional to the ratio v_{eq}/c, 2) the anomalous force acting on the spacecraft must change the λ component of the spacecraft's speed, and 3) the speed-change must be proportional to v_{in}.

The objective of this article is threefold: 1) derive a new causal version of classical acausal Newtonian theory, 2) show that this new version is able to produce exact agreement with all six of the anomalous speed-changes reported by Anderson *et al.*, and 3) show that it is also able to explain exactly a "lunar orbit anomaly" that will be described below. The proposed new version for Newtonian theory requires only mainstream physics: 1) classical Newtonian theory and 2) the causality principle which requires time-retardation of the gravitational field. It also satisfies the three requirements of the empirical prediction formula.

The proposed theory is based on a simple correction that converts Newton's acausal theory into a causal theory. The resulting causal theory has a new, previously overlooked, time-retarded transverse component, designated $\boldsymbol{g}_{\text{trt}}$, which depends on $1/c_{\text{g}}$, where c_{g} is the speed of gravity, which approximately equals the speed of light. The new total gravitational field for a large spinning sphere, $\boldsymbol{g}$, has two components, the standard well-known classical acausal radial component, $\boldsymbol{g}_{\text{r}}$, and a new relatively small time-retarded transverse vortex component, $\boldsymbol{g}_{\text{trt}}$. The total vector field $\boldsymbol{g} = \boldsymbol{g}_{\text{r}} + \boldsymbol{g}_{\text{trt}}$. The zero-divergence vortex transverse vector field $\boldsymbol{g}_{\text{trt}}$ is orthogonal to the irrotational radial vector field $\boldsymbol{g}_{\text{r}}$.

The new total vector field is consistent with Helmholtz's theorem, which states that any physical vector field can be expressed as the sum of the gradient of a zero-rotational scalar potential and the curl of a zero-divergence vector potential [3]. This means that $\boldsymbol{g}_{\rm r}$ can be derived in the standard way from the gradient of a scalar potential, and $\boldsymbol{g}_{\rm trt}$ can be derived from the curl of a vector potential, but $\boldsymbol{g}_{\rm trt}$ cannot be derived from the gradient of a scalar potential.

The proposed causal version can be derived by using the slow-speed weak-field approximation for general relativity theory.

2 Summary of the derivation of the formulas for the time-retarded transverse gravitational field and the predicted flyby speed-changes

In the section entitled *The Linear Approximation to GR* in W. Rindler's popular textbook *Essential Relativity* [4], Rindler derives the formulas for the time-retarded scalar potential φ, the time-retarded "gravitoelectric" acceleration field $\boldsymbol{e}$, the time-retarded vector potential $\boldsymbol{a}$, and the time-retarded "gravitomagnetic" induction field $\boldsymbol{h}$. His formulas for φ, $\boldsymbol{e}$, $\boldsymbol{a}$, and $\boldsymbol{h}$ are derived from general relativity theory by using the slow-speed weak-field approximation. They are as follows

$$\left.\begin{aligned} \varphi &= G \iiint \left[\frac{\rho}{r''}\right] dV, & \boldsymbol{a} &= \frac{G}{c} \iiint \left[\frac{\rho\,\boldsymbol{u}}{r''}\right] dV \\ \boldsymbol{e} &= -\boldsymbol{\nabla}\varphi, & \boldsymbol{h} &= \boldsymbol{\nabla}\times 4\,\boldsymbol{a} \end{aligned}\right\}, \quad (2)$$

where ρ is the mass-density of the central object, $\boldsymbol{u}$ is the inertial velocity (the velocity in an inertial frame) of a source-point-mass in the central object, $\boldsymbol{r}''$ is the vector distance from an inner source-point-mass to an outer field-point, and the square brackets [ ] mean that the enclosed function is to be evaluated at the retarded time, i.e., the time retarded by the light travel time from the source-point to the field-point.

Let the origin for an inertial (nonaccelerating and nonrotating) frame-of-reference coincide with the center-of-mass of a contiguous central object. Let $\boldsymbol{r}'$ be the radial vector from the origin to a source-point-mass in the central object, and let $\boldsymbol{r}$ be the radial vector from the origin to an external field-point, so that $\boldsymbol{r}'' = \boldsymbol{r} - \boldsymbol{r}'$. The square brackets in the triple integrals in (2) indicate that the integrands $[\rho/r'']$ and $[\rho\,\boldsymbol{u}/r'']$ are to be integrated over the volume of the central object at the retarded time.

Let m be the mass of a test-mass that occupies the field-point at $\boldsymbol{r}$, and let $\boldsymbol{v}$ be the inertial velocity of the test mass. The analogous Lorentz force law, i.e., the formula for the time-retarded gravitational force $\boldsymbol{F}$ acting on m at $\boldsymbol{r}$, is [4]

$$\begin{aligned} \boldsymbol{F} = -m\left(\boldsymbol{e} + \frac{1}{c}(\boldsymbol{v}\times\boldsymbol{h})\right) &= -m\,\boldsymbol{\nabla}\left(G\iiint\left[\frac{\rho}{r''}\right] dV\right) - \\ &-m\left(\boldsymbol{v}\times\left(\boldsymbol{\nabla}\times\left(\frac{4G}{c^2}\iiint\left[\frac{\rho\,\boldsymbol{u}}{r''}\right] dV\right)\right)\right). \quad (3) \end{aligned}$$

Rindler's time-retarded version for the slow-speed weak-field approximation gives a complete stand-alone time-retarded solution. The time-retarded fields were derived from general relativity theory, but there is no need for further reference to the concepts and techniques of general relativity theory. Needed concepts and techniques are those of classical Newtonian theory.

Furthermore, Rindler's formulas satisfy the causality principle because the fields are time-retarded. Rindler's version gives a good first approximation only if

$$v^2 \ll c^2, \qquad u^2 \ll c^2, \qquad \frac{GM}{r} = |\varphi| \ll c^2, \quad (4)$$

where M is the total mass of the central object.

Notice in (3) that the acceleration caused by the gravitoelectric field $\boldsymbol{e}$ is independent of c, but the acceleration caused by the gravitomagnetic induction field $\boldsymbol{h}$ is reduced by the factor $1/c^2$. The numerical value for c is on the order of 3×10^8 m/s. If the magnitude for $\boldsymbol{e}$ is on the order of 10 m/s^2 (the Earth's field at the surface), and the magnitudes for $\boldsymbol{u}$ and $\boldsymbol{v}$ are on the order of 10^4 m/s, the relative magnitude for the acceleration caused by $\boldsymbol{h}$ would be on the order of $10\times4\,(10^4/3\times10^8)^2m/s^2 \sim 10^{-8}$m/s^2. This estimate shows that, for slow-speed weak-field practical applications in the real world, the acceleration caused by $\boldsymbol{h}$ is totally negligible compared to the acceleration caused by $\boldsymbol{e}$.

The empirical formula indicates that the flyby speed-change is reduced by $1/c$, not by $1/c^2$, which rules out the gravitomagnetic field as a possible cause for the flyby anomalies. The acceleration of $\boldsymbol{h}$ is simply too small to explain the flyby anomalies.

Consequently, the practicable version for Rindler's Lorentz force law becomes the same as a time-retarded version for Newton's well-known inverse-square law

$$\boldsymbol{F} = -Gm\,\boldsymbol{\nabla}\iiint\left[\frac{\rho}{r''}\right] dV, \quad (5)$$

where $\boldsymbol{F}$ is the time-retarded gravitational force acting on m.

Let $d^3\boldsymbol{F}$ be the time-retarded elemental force of an elemental point-mass source dm'. The time-retarded version for Newton's inverse-square law becomes

$$d^3\boldsymbol{F} = -Gm\,\frac{dm'}{r''^2}\,\frac{\boldsymbol{r}''}{r''}, \quad (6)$$

where $\boldsymbol{r}''/r''$ is a unit vector directed towards increasing $\boldsymbol{r}''$.

By definition, the gravitational field of a source at $\boldsymbol{r}'$ is the gravitational force of the source dm' that acts on a test-mass m at $\boldsymbol{r}$ per unit mass of the test-mass. The traditional symbol for the Newtonian gravitational field is $\boldsymbol{g}$. Therefore, the formula for the time-retarded elemental gravitational field $d^3\boldsymbol{g}$ of an elemental point-mass-source at $\boldsymbol{r}'$ for a field-point at $\boldsymbol{r}$ becomes

$$d^3\boldsymbol{g} = \frac{d^3\boldsymbol{F}}{m} = -G\,\frac{dm'}{r''^2}\,\frac{\boldsymbol{r}''}{r''}. \quad (7)$$

The negative sign indicates that the gravitational force is attractive.

Let t be the observed coordinate time at $\boldsymbol{r}$, let t' be the retarded time at $\boldsymbol{r}'$, and let c_g be the speed of propagation of the gravitational field. The connection between t and t' is

$$t = t' + \frac{r''}{c_g}, \quad t' = t - \frac{r''}{c_g}. \tag{8}$$

Obviously, $t(t')$ is a function of t', and *vice versa*, $t'(t)$ is a function of t. The Jacobian for the transformation from t to t' is given by

$$\text{Jacobian} = \frac{dt}{dt'} = 1 + \frac{1}{c_g}\frac{dr''}{dt'}. \tag{9}$$

This Jacobian is needed to solve the triple integral over the volume of the central object. It leads to the necessary factor $1/c_g$, where c_g is the speed of propagation of the Earth's gravitational field [5].

Let $\rho(\boldsymbol{r}')$ be the mass-density of the central object at $\boldsymbol{r}'$. Then

$$dm' = \rho(\boldsymbol{r}')\,dV. \tag{10}$$

The resulting formula for the elemental total gravitational field $d^3\boldsymbol{g}$, which consists of the radial component $d^3\boldsymbol{g}_r$ and the transverse component $d^3\boldsymbol{g}_{trt}$, becomes $d^3\boldsymbol{g} = d^3\boldsymbol{g}_r + d^3\boldsymbol{g}_{trt}$. The differential formulas for each component become

$$d^3\boldsymbol{g}_r = -G\frac{dm'}{r''^2}\left(\frac{\boldsymbol{r}''}{r''}\right)_r, \qquad d^3\boldsymbol{g}_{trt} = -G\frac{dm'}{r''^2}\left(\frac{\boldsymbol{r}''}{r''}\right)_{trt}, \tag{11}$$

where $(\boldsymbol{r}''/r'')_r$ is the radial component of the unit vector and $(\boldsymbol{r}''/r'')_{trt}$ is the transverse component of the unit vector. The total field is obtained by a triple integration over the volume of the central object at the retarded time.

Let (X, Y, Z) be the rectangular coordinates for the inertial frame-of-reference, and let the Z-axis coincide with the spin axis of the central object. Let RC be the relative radial component, and let TC_Z be the magnitude for the Z-axis component of the relative transverse component. As can be seen in Fig. 1, the formulas for RC and TC_Z are related to $\boldsymbol{r}$, $\boldsymbol{r}'$, and $\boldsymbol{r}''$ by

$$\left.\begin{aligned} RC &= \frac{\boldsymbol{r}\cdot\boldsymbol{r}''}{r''r} = \frac{\boldsymbol{r}\cdot\boldsymbol{r} - \boldsymbol{r}\cdot\boldsymbol{r}'}{r''r} \\ TC_Z &= \frac{(\boldsymbol{r}\times\boldsymbol{r}'')_Z}{r''r} = \frac{(\boldsymbol{r}'\times\boldsymbol{r})_Z}{r''r} = \frac{r'_X r_Y - r'_Y r_X}{r''r} \end{aligned}\right\}, \tag{12}$$

where r_X, r_Y are the X, Y components of $\boldsymbol{r}$, and r'_X, r'_Y are the X, Y components of $\boldsymbol{r}'$.

The formula for the magnitude of $\boldsymbol{g}_{trt}$ becomes [5]

$$\boldsymbol{g}_{trt} = \iiint\left(-G\frac{dm'}{r''^2}\right)(TC_Z)\,(\text{Jacobian}). \tag{13}$$

The triple integral is rather easy to solve by using numerical integration if the central object can be approximated by a large spinning isotropic sphere. To get a good first approximation, the Earth was simulated in [5] by a large spinning isotropic sphere.

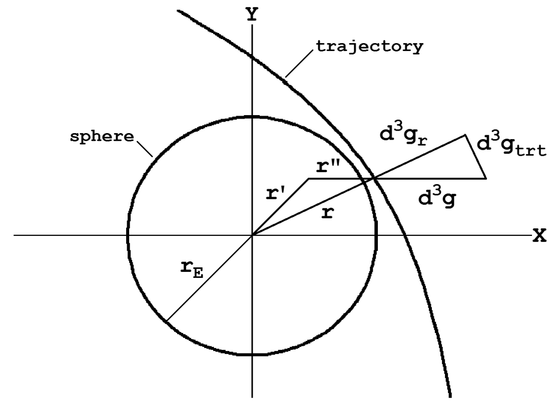

Fig. 1: Depiction of the vector distances $\boldsymbol{r}$, $\boldsymbol{r}'$, and $\boldsymbol{r}''$ and the components of the vector field $d^3\boldsymbol{g}$, $d^3\boldsymbol{g}_r$, and $d^3\boldsymbol{g}_{trt}$.

The formulas for the geocentric radial distance to the field-point and its derivative are

$$\left.\begin{aligned} r(\theta) &= \frac{r_p(1+\varepsilon)}{1+\varepsilon\cos\theta} \\ \frac{dr}{d\theta} &= \frac{r(\theta)^2}{r_p}\frac{\varepsilon}{1+\varepsilon}\sin\theta \end{aligned}\right\}, \tag{14}$$

where θ is the parametric polar coordinate angle for the spacecraft in the plane of the trajectory, r_p is the geocentric radial distance at perigee, and ε is the eccentricity of the trajectory.

It is shown in [5] that the formula for the Jacobian is

$$\begin{aligned} \text{Jacobian} &= 1 + \frac{1}{c_g}\frac{dr''}{dt} = \\ &= 1 - \frac{r}{c_g}\frac{r'}{r''}\left(\Omega_\phi - \Omega_E\right)\cos\lambda'\,\sin\phi'. \end{aligned} \tag{15}$$

It is also shown in [5] that the only term for $d^3\boldsymbol{g}_{trt}$ that will survive the triple integration is

$$d^3 g_{trt} = -G\bar{\rho}\,r_E\frac{\Omega_\phi - \Omega_E}{\Omega_E}\cos^2(\lambda)\,IG\,\frac{dr'}{r_E}\,d\lambda'\,d\phi', \tag{16}$$

where $\bar{\rho}$ is the mean value for $\rho(r')$ and the formula for the integrand is

$$IG = \frac{r_E^3}{r^3}\frac{\rho(r')}{\bar{\rho}}\frac{r'^4}{r_E^4}\cos^3(\lambda')\frac{\sin^2\phi'}{(1+x)^2} \tag{17}$$

where the variable x is defined by

$$x \equiv \frac{r'^2}{r^2} - 2\frac{r'}{r}\cos\lambda'\,\cos\phi'. \tag{18}$$

It has been shown in [5] that the resulting formula for the magnitude of the transverse component is

$$g_{trt}(\theta) = -G\frac{I_E}{r_E^4}\frac{v_{eq}}{c_g}\frac{\Omega_\phi(\theta) - \Omega_E}{\Omega_E}\cos^2(\lambda(\theta))\,PS(r(\theta)), \tag{19}$$

where G is the gravity constant, I_E is the Earth's spherical moment of inertia, r_E is the Earth's spherical radius, Ω_E is the Earth's spin angular speed, υ_{eq} is the Earth's equatorial surface speed, c_g is the speed of propagation of the Earth's gravitational field, θ is the spacecraft's parametric polar coordinate angle in the plane of the orbit or trajectory, $\Omega_\theta = d\theta/dt$ is the spacecraft's angular speed, Ω_ϕ is the azimuthal ϕ-component of Ω_θ, λ is the spacecraft's geocentric latitude, r is the spacecraft's geocentric radial distance, and $PS(r)$ is an inverse-cube power series representation for the triple integral over the Earth's volume. The formula for $PS(r)$ is [5]

$$PS(r) \equiv \left(\frac{r_E}{r}\right)^3 \left(C_0 + C_2\left(\frac{r_E}{r}\right)^2 + C_4\left(\frac{r_E}{r}\right)^4 + C_6\left(\frac{r_E}{r}\right)^6\right),$$

where the values for the coefficients are

$$C_0 = 0.50889, \quad C_2 = 0.13931,$$
$$C_4 = 0.01013, \quad C_6 = 0.14671.$$

If the magnitude is negative, i.e., if $\Omega_\phi > \Omega_E$ (prograde), the vector field component $\boldsymbol{g}_{trt}$ is directed towards the east. If $\Omega_\phi < 0$ (retrograde), it is directed towards the west.

The formula for the time-retarded transverse gravitational field, g_{trt}, satisfies the first requirement of the empirical prediction formula. It is proportional to $\upsilon_{eq}/c_g \cong \upsilon_{eq}/c$. But the empirical prediction formula also requires that the speed-change must be in the λ-component of the spacecraft's velocity, $\boldsymbol{\upsilon}_\lambda$. The magnitude for the λ-component is defined by

$$\upsilon_\lambda = r_\lambda \frac{d\lambda}{dt} = r_\lambda \frac{d\lambda}{d\theta}\frac{d\theta}{dt} = r_\lambda \Omega_\theta \frac{d\lambda}{d\theta}, \qquad (20)$$

where r_λ is the λ-component of r.

The velocity component, $\boldsymbol{\upsilon}_\lambda$, is orthogonal to $\boldsymbol{g}_{trt}$. Consequently, $\boldsymbol{g}_{trt}$ cannot directly change the magnitude of $\boldsymbol{\upsilon}_\lambda$ (it changes the direction).

However, a hypothesized induction-like field, designated $\boldsymbol{F}_\lambda$, can be directed perpendicularly to $\boldsymbol{g}_{trt}$ in the $\boldsymbol{\upsilon}_\lambda$-direction. Assume that the ϕ-component of the curl of $\boldsymbol{F}_\lambda$ equals $-k\,d\boldsymbol{g}_{trt}/dt$, where k is a constant. This induction-like field can cause a small change in the spacecraft's speed. The reciprocal of the constant k, $\upsilon_k = 1/k$, called the "induction speed", becomes an adjustable parameter for each case. The average for all cases gives an overall constant for the causal theory.

The formula for the magnitude of $\boldsymbol{F}_\lambda$ has been shown in [5] to be

$$F_\lambda = \frac{\upsilon_{eq}}{\upsilon_k}\frac{r_E}{r(\theta)}\int_0^\theta \frac{r(\theta)}{r_E}\frac{\Omega_\theta(\theta)}{\Omega_E}\frac{1}{r_E}\frac{dr}{d\theta}\frac{dg_{trt}}{d\theta}\,d\theta. \qquad (21)$$

The acceleration caused by $\boldsymbol{F}_\lambda$ satisfies the second requirement of the empirical prediction formula, the one that requires the anomalous force to change the λ-component of the spacecraft's velocity.

The anomalous time rate of change in the spacecraft's orbital energy is given by the dot product, $\boldsymbol{\upsilon} \cdot \boldsymbol{F}_\lambda$. It has been shown in [5] that the calculated asymptotic speed-change, $\delta\upsilon_{trt}$, is given by

$$\delta\upsilon_{trt} = \delta\upsilon_{in} + \delta\upsilon_{out}, \qquad (22)$$

where

$$\delta\upsilon_{in} = \delta\upsilon(\theta_{min}), \qquad \delta\upsilon_{out} = \delta\upsilon(\theta_{max}), \qquad (23)$$

and

$$\delta\upsilon(\theta) = \frac{\upsilon_{in}}{2}\int_0^\theta \frac{r_\lambda(\theta)\,F_\lambda(\theta)}{\upsilon_{in}^2}\frac{d\lambda}{d\theta}\,d\theta. \qquad (24)$$

The angles θ_{min} and θ_{max} are the minimum and maximum values for θ. The initial speed $\upsilon_{in} = \upsilon(\theta_{min})$. The speed-change $\delta\upsilon(\theta)$ is proportional to υ_{in}, which satisfies the third requirement of the empirical prediction formula.

3 Summary of the change in the Moon's orbital speed caused by the Earth's time-retarded transverse gravitational field

In 1995, F. R. Stephenson and L. V. Morrison published a remarkable study of records of eclipses from 700 BC to 1990 AD [6]. They conclude*: 1) the LOD has been increasing on average during the past 2700 years at the rate of $+1.70 \pm 0.05$ ms cy^{-1} (i.e. $(+17.0 \pm 0.5) \times 10^{-6}$ s per year), 2) tidal braking causes an increase in the LOD of $+2.3 \pm 0.1$ ms cy^{-1} (i.e. $(+23 \pm 1) \times 10^{-6}$ s per year), and 3) there is a non-tidal decrease in the LOD, numerically -0.6 ± 0.1 ms cy^{-1} (i.e. $(-6 \pm 1) \times 10^{-6}$ s per year).

Stephenson and Morrison state that the non-tidal decrease in the LOD probably is caused by post-glacial rebound. Post-glacial rebound decreases the Earth's moment of inertia, which increases the Earth's spin angular speed, and thereby decreases the LOD. But post-glacial rebound cannot change the Moon's orbital angular momentum.

According to Stephenson and Morrison, tidal braking causes an increase in the LOD of $(23 \pm 1) \times 10^{-6}$ seconds per year, which causes a decrease in the Earth's spin angular momentum, and by conservation of angular momentum causes an increase in the Moon's orbital angular momentum. It has been shown in [5] that tidal braking alone would cause an increase in the Moon's orbital speed of $(19 \pm 1) \times 10^{-9}$ m/s per year, which corresponds to an increase in the radius of the Moon's orbit of (14 ± 1) mm per year.

But lunar-laser-ranging experiments have shown that the radius of the Moon's orbit is actually increasing at the rate of (38 ± 1) mm per year [7]. This rate for increase in the radius corresponds to an increase in the orbital speed of $(52 \pm 2) \times 10^{-9}$ m/s per year. Clearly there is an unexplained or anomalous difference in the change in the radius of the orbit of (-24 ± 2) mm per year $(38 - 14 = 24)$, and a corresponding anomalous difference in the change in the orbital speed of

*LOD means length-of-solar-day and ms cy^{-1} means milliseconds per century.

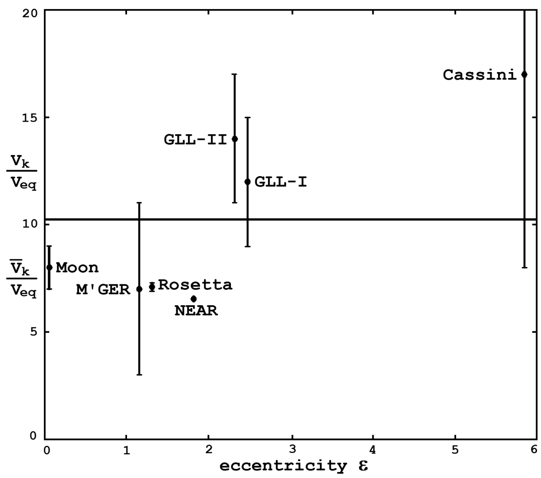

Fig. 2: Required induction speed ratio (designated by •), υ_k/υ_{eq} ± an estimate for the uncertainty, versus eccentricity ε. The mean value for all seven ratios is shown by the horizontal line.

$(-33 \pm 3) \times 10^{-9}$ m/s per year $(52 - 19 = 33)$. This "lunar orbit anomaly" cannot be caused by post-glacial rebound, but it can be caused by the proposed causal version of Newton's theory.

It has been shown in [5] that the causal version of Newton's theory produces a change in the Moon's orbital speed of $(-33 \pm 3) \times 10^{-9}$ m/s per year if the value for the induction speed $\upsilon_k = (8 \pm 1)\,\upsilon_{eq}$. Therefore, the causal theory explains exactly the lunar orbit anomaly. It gives a new closed orbit case for anomalous speed-changes.

4 Calculated speed-changes caused by the time-retarded version of Newton's theory

It has been shown in [5] that the causal version of Newton's theory explains exactly the six flyby anomalies reported by Anderson *et al*. [1]. The required values for υ_k cluster between $6\,\upsilon_{eq}$ and $17\,\upsilon_{eq}$.

A graph of the required induction speed ratios, υ_k/υ_{eq}, versus eccentricity ε, Fig. 2, shows that the required value for υ_k for the lunar orbit anomaly is consistent with the required values for υ_k for the six Earth flyby anomalies. The average ± standard deviation is

$$\bar{\upsilon}_k = (10 \pm 4)\,\upsilon_{eq} = 5 \pm 2 \text{ km/s}. \qquad (25)$$

It will be interesting to compare this average, 5 ± 2 km/s, with parameter values for other flyby theories.

5 Predicted speed-changes for future high-precision Doppler-shift experiments

The speed-change caused by the causal version of Newton's theory depends on the speed of gravity c_g, the properties of the central sphere; mass M_E, radius r_E, angular speed Ω_E, moment of inertia I_E, and equatorial surface speed υ_{eq}, on the orbital properties of the spacecraft; radius at perigee r_p, eccentricity ε, inclination to the equatorial plane α_{eq}, and latitude at perigee λ_p, and the induction speed υ_k. If $\varepsilon = 0$ or if $\alpha_{eq} = 0$, the speed-change $\delta\upsilon_{trt} = 0$. Even if $\varepsilon \neq 0$ and $\alpha_{eq} \neq 0$, $\delta\upsilon_{trt}$ is still equal to zero if perigee is over the equator ($\lambda_p = 0°$) or one of the poles ($\lambda_p = \pm 90°$). The maximum speed-change occurs for spacecrafts in highly eccentric and inclined near-Earth orbits.

r_p/r_E	P (hours)	$\delta\upsilon_{yr}$ (mm/s per year)	$\delta\upsilon r_{yr}$ (mm/s per year)
2	11.2	+315	−517
3	20.7	+29.5	−76.8
4	31.8	+3.93	−21.0
5	44.4	+0.173	−7.97
6	58.4	−0.422	−3.69
7	73.6	−0.442	−1.95
8	89.9	−0.362	−1.14

Table 1: Calculated period P (in hours) and predicted speed-change for prograde orbits $\delta\upsilon_{yr}$ (in mm/s per year), and the predicted speed-change for retrograde orbits $\delta\upsilon r_{yr}$ (in mm/s per year), for a spacecraft in a near-Earth orbit with $\varepsilon = 0.5$, $\alpha_{eq} = 45°$, $\lambda_p = 45°$, $\upsilon_k = 14\,\upsilon_{eq}$, and for r_p ranging from $2\,r_E$ to $8\,r_E$ [5].

Assume $c_g = c$ and the induction speed is its largest probable value, $\upsilon_k = 14\,\upsilon_{eq}$. Suppose the orbital properties for a spacecraft are $\varepsilon = 0.5$, $\alpha_{eq} = 45°$, and $\lambda_p = 45°$. Let r_p range from $2\,r_E$ to $8\,r_E$. The period P is given by Kepler's 3rd law, and the annual speed change for prograde $\delta\upsilon_{yr} = N_{rev}\,\delta\upsilon_{trt}$, and for retrograde $\delta\upsilon r_{yr} = N_{rev}\,\delta\upsilon_{trt}$, where N_{rev} is the number of revolutions per year. Calculated speed-changes are listed in Table 1 [5].

6 Other theories which explain the Earth flyby anomalies

There are at least two other published theories that explain the Earth flyby anomalies: 1) the 3-space flow theory of R. T. Cahill [8], and 2) the exponential radial field theory of H. J. Busack [9].

In [8] Cahill reviews numerous Michelson interferometer and one-way light-speed experiments which clearly show an anisotropy in the velocity of light. His calculated flyby speed-changes depend on the direction and magnitude for 3-space inflow at the spacecraft on the date and time of the flyby. Cahill found that the average speed for 3-space inflow is 12±5 km/s. Cahill's average, 12±5 km/s, essentially equals the average value for υ_k (25), 5±2 km/s.

In [9] Busack applies a small exponential correction for the Earth's radial gravitational field. If $f(\boldsymbol{r}, \boldsymbol{\upsilon})$ is Busack's correction, the inverse-square law becomes

$$\boldsymbol{g}_r(\boldsymbol{r}, \boldsymbol{\upsilon}) = -\frac{GM_E}{r^2}\frac{\boldsymbol{r}}{r}\left(1 + f(\boldsymbol{r}, \boldsymbol{\upsilon})\right),$$

where $f(\boldsymbol{r}, \boldsymbol{v})$ is expressed as

$$f(\boldsymbol{r}, \boldsymbol{v}) = A \exp\left(-\frac{r - r_E}{B - C(\boldsymbol{r} \cdot \boldsymbol{v})/(\boldsymbol{r} \cdot \boldsymbol{v}_{Sun})}\right).$$

The velocity $\boldsymbol{v}$ is the velocity of the field-point in the "gravitational rest frame in the cosmic microwave background", and $\boldsymbol{v}_{Sun}$ is the Sun's velocity in the gravitational rest frame. Numerical values for the adjustable constants are approximately $A = 2.2 \times 10^{-4}$, $B = 2.9 \times 10^5$ m, and $C = 2.3 \times 10^5$ m. Busack found that these values produce rather good agreement with the observed values for the flyby speed-changes.

Both of these alternative theories require a preferred frame-of-reference. Neither has been tested for the lunar orbit anomaly, and neither satisfies the causality principle because neither depends on the speed of gravity.

7 Conclusions and recommendations

This article shows conclusively that the proposed causal version of Newton's theory agrees with the now-known facts-of-observation. It applies only for slow-speeds and weak-fields. Effects of time retardation appear at the relatively large first-order v/c_g level, but they have not been seen in the past because they decrease inversely with the cube of the closest distance. If perigee is very close, however, time retardation effects can be relatively large. It is recommended that various available methods be used to detect new observations of effects of the causality principle.

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